A FEMU methodology to obtain material parameters for thermoplastic materials

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Abstract

Necking formation on thermoplastic materials presents a particular heterogeneous strain field that, as pointed out by many authors, may mask the real mechanical response of the material. The necking may propagate to the entire specimen when submitted to finite strain. Moreover, the mechanical response of thermoplastics is highly dependent of the strain rate. These issues incorporate difficulties on the identification parameter procedure. Usually, only the force information from a monotonic experimental testing may be not sufficient to characterize the mechanical response of the material and the kinematic behavior of the necking region. Then, in order to determine the material parameters for a constitutive model capable to represent the realistic behavior of these materials, suitable characterization techniques should be employed. In this work, a numerical-experimental characterization is presented to obtain material parameters from a nonlinear thermoplastic submitted to large strains. The proposed characterization is based on a Finite Element Method Updating (FEMU) methodology that take into account the experimental mechanical response of force, obtained from a uniaxial tensile test, and the localized displacements of the necking region, obtained from optical measurements. Experimental tests are performed on a polyvinyl chloride (PVC) specimen in order to obtain the typical force and displacement curves. Furthermore, Digital Image Correlation (DIC) technique is used to provide the displacement field from the necking region. A constitutive model for nonlinear elastoplastic materials is used providing the numerical mechanical response of force and the displacement field on the necking region. The numerical and experimental data of force and localized displacement are used to define a particular objective function. Finally, the parameters of the material model are determined by an optimization procedure using a hybrid methodology that combines genetic and gradient based algorithms. The results shown that this methodology is capable to take into account force and kinematic responses into an identification parameters procedure. Also, for the material models studied, only the force data is not capable to capture the kinematic behavior observed experimentally, pointing out that experimental information of the necking region is necessary.

Keywords: FEMU, constitutive parameter identification, thermoplastics, finite strain

1 Introduction

Thermoplastic materials present good physical properties and low manufacturing costs, being attractive to the industry and used in diverse applications such as electronic devices, clothing and vehicles. However, applications where structural evaluation is needed, the usage of thermoplastics is still restrict, because of the highly nonlinear behavior when the material is submitted to high mechanical stress.

Thermoplastics submitted to finite strains may present diverse mechanical behaviors like viscoplasticity, temperature sensivity, damage and anisotropy [1]. The wide range of phenomena that thermoplastics present under finite strains can pose difficulties in the characterization process and constitutive model selection.

Tensile testing of thermoplastics submitted to finite strains may also present heterogeneous strains fields in particular regions, due to necking and cold-drawing (necking propagation) which may mask the actual stress-strain curve measured by a tensile testing machine, turning the real stress-strain curve characterization in a non-trivial procedure [2]. Thus, constitutive characterization using force response only, from a tensile testing, may not be sufficient due to the necking development that governs the stress response.

An approach to provide additional data to the mechanical characterization can be performed using kinematic field measurements from the necking region of the specimen. These kinematic measurements
can be obtained using DIC [3] technique, which is able to determine displacement fields without direct physical contact with the specimen using frames captured from the studied region during the testing.

Thereby, to study the mechanical characterization problem, in this work a tensile testing was performed on a PVC specimen to determine its force and displacement field from the necking region. Experimental data obtained was used in a FEMU [4] methodology with force and displacement objective functions [5] to determine optimal constitutive parameters of a multilinear constitutive model using a hybrid optimization procedure combining genetics and gradient based algorithms.

2 Experimental data

Experimental tensile testing used a PVC specimen with dog-bone shape geometry, where a machined indentation in the middle of the specimen was made. Specimen principal dimensions are shown in figure 1. The indentation was made to promote necking formation and propagation in a predetermined position. A clip gauge, with a gauge length of 50 mm, was installed to determine the displacement applied in the testing near to the necking region.

![Figure 1: PVC specimen dimensions](image)

Tensile testing was performed at constant velocity of 5 mm/min relative grip speed. Force response was obtained from a load cell and displacement field from the necking region was determined by DIC. To promote image correlation the specimen was painted with black and white random speckles, as shown in figure 2. Displacement responses employed in the characterization procedure corresponded to the relative displacement of markers A, B and C to marker M, in the y-direction of figure 2.

![Figure 2: DIC markers](image)

3 Inverse problem

FEMU methodology was employed to solve the inverse problem of determining the optimal sets of constitutive parameters of the constitutive model studied. The methodology employed sought to minimize the difference between the experimental data and the numerical data from a FEM model, which corresponds to the experimental testing performed. FEMU procedure employed in this work is described by
Objective functions

\begin{align*}
  f^F(F^{\text{exp}}, F^{\text{num}}(m)) &= \sum_{i=1}^{\text{steps}} \left( \frac{F_i^{\text{exp}} - F_i^{\text{num}}(m)}{F_{\text{max}}} \right)^2 \\
  f^d(d^{\text{exp}}, d^{\text{num}}(m)) &= \sum_{j=1}^{\text{mrk}} \left( \sum_{i=1}^{\text{steps}} \left( \frac{d_{ji}^{\text{exp}} - d_{ji}^{\text{num}}(m)}{d_{ji,\text{max}}^{\text{exp}}} \right)^2 \right)
\end{align*}

where exp and num superscripts refers respectively to experimental and numerical data, \( f^F \) is the force objective function and \( f^d \) is the displacement objective function. \( F_i \) corresponds to the force response and \( d_i \) for the relative transverse displacements from the positions of the markers of figure 2. Index \( i \) refers to each time step evaluated and index \( j \) refers to the three relative displacements studied in the characterization process.

To determine the optimal sets of constitutive parameters, the objective functions presented in Eq.(1) and Eq.(2) were minimized separately. Genetic algorithms were used to determine adequate initial values for each objective function, these initial values were then employed to determine the optimal sets of constitutive parameters using the Levenberg-Marquardt algorithm [6].

4 Constitutive model

The constitutive model employed in this work was a multilinear isotropic [7] model from ANSYS software. This model has linear elastic behavior and in the plastic region uses an yield function \( Q \),

\[ Q = \sigma_e(\sigma) - \sigma_y(\kappa) \]

where \( \sigma_e \) is an equivalent scalar stress, \( \sigma_y \) is the current yield stress from the problem, \( \sigma \) is the Cauchy stress tensor and \( \kappa \) is the plastic work. The plastic work rate is given as follows,

\[ \dot{\kappa} = \sigma : \dot{\varepsilon}^P \]

where \( \varepsilon^P \) is a plastic strain tensor. This model have multilinear denomination due to the multilinear behavior of the stress-plastic work curve (figure 4a) and isotropic denomination because the current yield

\[ Q = \sigma_e(\sigma) - \sigma_y(\kappa) \]
stress surface keeps the same centerline during yielding. A plastic flow rule is presented to define the plastic evolution,

$$\dot{\varepsilon}^p = \lambda \frac{\partial Q}{\partial \sigma}$$

(5)

where $\lambda$ is a plastic multiplier. Finally, a set of loading and unloading conditions can be defined,

$$\lambda \geq 0 \quad Q \leq 0 \quad Q\lambda = 0 \quad \dot{Q}\lambda = 0$$

(6)

The constitutive model used has in total ten constitutive parameters. Two parameters correspond to the elastic region, which are the elastic modulus ($E$) and the Poisson coefficient ($\nu$). The other eight constitutive parameters correspond to the plastic region, as shown in figure 4a, are yield stress $\sigma_0$, linear stiffening $H$, plastic strain increment $\epsilon^{ps}$, second linear stiffening $I$, stress increment $\Delta_1$, stiffening increment $\phi_1$, second stress increment $\Delta_2$ and second stiffening increment $\phi_2$. A stress-strain curve sample of the multilinear model is presented in figure 4b.

![Stress-plastic work behavior](image1)

![Stress-strain curve behavior](image2)

(a) Stress-plastic work behavior  (b) Stress-strain curve behavior

Figure 4: Applied multilinear model behavior

5 Results and discussion

In this section the optimal results obtained for each objective function are presented. Optimal constitutive parameters and objective function values of the force objective function ($f_F$) are shown in table 1, force and relative displacement results are shown in figure 5 and the optimal stress-strain curve is shown in figure 7a. Optimal results from the force objective show that the numerical force response was able to reproduce the experimental force behavior, however the numerical response was unable to represent adequately the experimental relative displacement from any of the studied markers.

Table 1: Constitutive parameters and objective function values of the minimum force objective function

<table>
<thead>
<tr>
<th>$E$</th>
<th>$\nu$</th>
<th>$\sigma_0$</th>
<th>$H$</th>
<th>$\epsilon^{ps}$</th>
<th>$I$</th>
<th>$\Delta_1$</th>
<th>$\phi_1$</th>
<th>$\Delta_2$</th>
<th>$\phi_2$</th>
<th>$f_F$</th>
<th>$f_d$</th>
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<tr>
<td>2848.6</td>
<td>0.43</td>
<td>37.8</td>
<td>0.0</td>
<td>0.505</td>
<td>110.8</td>
<td>14.8</td>
<td>21.6</td>
<td>0.049</td>
<td>1.25</td>
<td>0.275</td>
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</table>

For the case of optimal results using the displacement objective function ($f_d$), constitutive parameters and objective function values are presented in table 2, force and relative displacement results are presented in figure 6 and the optimal stress-strain curve is shown in figure 7b. From the results of the optimal displacement objective, the numerical response was unable to represent both force, and relative displacements, of the experimental response. Thereby, both objective functions studied (force and displacement objective functions) were unable to determine a set of constitutive parameters to represent the experimental relative displacement responses.

The stress-strain curves obtained from the force and displacement objective functions studied have also strong dissimilar behavior. Force objective function present a stress-strain curve, shown in figure 7a, with a plateau between 50 and 60 MPa and a smooth transition between the elastic and plastic region.
Figure 5: Minimum force objective function ($f^F$) responses of force and relative displacement

Figure 6: Minimum displacement objective function ($f^d$) responses of force and relative displacement

Figure 7: Optimal stress-strain curves
Table 2: Constitutive parameters and objective function values of the minimum displacement objective function

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>ν</th>
<th>σ0</th>
<th>H</th>
<th>εps</th>
<th>I</th>
<th>Δ₁</th>
<th>φ₁</th>
<th>Δ₂</th>
<th>φ₂</th>
<th>f’</th>
<th>f”</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>2394.4</td>
<td>0.38</td>
<td>28.42</td>
<td>0.0</td>
<td>0.706</td>
<td>1338.9</td>
<td>10.9</td>
<td>33.6</td>
<td>32.2</td>
<td>10.8</td>
<td>558.3</td>
<td>0.0863</td>
</tr>
</tbody>
</table>

Stress-strain curve of the displacement objective function (figure 7b) has a plateau between 70 and 80 MPa and without a smooth transition between the elastic and plastic region.

6 Conclusions

In this work a numerical-experimental methodology was employed to characterize a multilinear constitutive model, using experimental data from a PVC specimen submitted to tensile testing under finite strains. A FEMU methodology was used with a mixed optimization procedure to minimize force and relative displacement objective functions and determine optimal sets of constitutive parameters.

Numerical force responses obtained using the set of constitutive parameters from the minimum of the force objective function were able to represent only the experimental force response, and the necking kinematic was not well represented. On the other hand the displacements responses obtained minimizing the displacement objective were unable to represent any of the experimental responses adequately.

Numerical analysis should present force and displacement responses in agreement with experimental observations in order to consider the constitutive model adequacy to represent the mechanical behavior of the material at the strain level studied. Then, in order to represent the experimental responses observed in this work, more complex constitutive models must be employed. Another important conclusion of this work is that the force response only may not be enough to guarantee the correct mechanical characterization.

7 References


