Adjoint based multi-objective shape optimization of a transonic airfoil under uncertainties

D. Kumar\(^1\), J. Miranda\(^1\), M. Raisee\(^2\), C. Lacor\(^1\)

\(^1\)Fluid Mechanics and Thermodynamics Research Group, Department of Mechanical Engineering, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, BELGIUM
\(^2\)School of Mechanical Engineering, College of Engineering, University of Tehran, P.O.Box: 11155-4563, Tehran, IRAN

e-mail: dinesh.kumar@vub.ac.be, joao.miranda@vub.ac.be, mraisee@ut.ac.ir, chris.lacor@vub.ac.be

Abstract

In the present paper, the results of ongoing work within the FP7 project UMRIDA are presented and discussed. The main idea of this work is to combine the non-intrusive polynomial chaos based uncertainty quantification methods with the gradient based methods for stochastic optimization. When introducing uncertainties in a design process, the objective is no longer deterministic and can be characterized by its mean and its variance, i.e. in a robust design the optimization becomes multi-objective. Gradient based optimization of the mean objective and of the variance of the objective therefore requires the gradient of both quantities. The gradient of the mean objective is combined with the gradient of its variance using weights. By changing the weights, the Pareto front (if any, i.e. if the 2 objectives are conflicting) or at least part of it can be recovered. The proposed method is applied to the optimal shape design of the transonic RAE2822 airfoil under uncertainties. In the current work, the flight conditions (the Mach number and the angle of attack) are considered as uniformly distributed uncertain parameters. The objectives considered are the mean drag coefficient and its standard deviation. In this work, the adjoint solver and the CFD solver of SU2 (an open source CFD solver) are coupled with the polynomial chaos methods for the optimal shape design of a transonic airfoil. Hicks-Henne functions are employed to parameterize the airfoil and to represent a new geometry in the design process. The optimization procedure is performed using the sequential least square programming (SLSQP) algorithm. Two optimal designs are obtained by considering two different points on the Pareto front.

Keywords: Robust optimization, adjoint based methods, polynomial chaos, shape optimization, uncertainties

1 Introduction

Over the last decade, with increasing computational resources and hardware power, design optimization is receiving more and more interest in aeronautical applications. Almost all computational models of a real world application contain uncertainties in physical properties, model parameters, initial conditions, boundary conditions and geometry, which in turn, make the solution uncertain or stochastic. These uncertainties always question the accuracy of the results and can alter the final design dramatically. To achieve a certain level of reliability in the final design, it is necessary to include the influence of the input uncertainties in the design process.

Conventional optimization deals with the problem of finding numerically minimum (or maximum) of an objective function. Robust or stochastic optimization is an extension of conventional optimization when uncertainties are also introduced in the design procedure. Introducing uncertainties in the design process, the objective becomes non-deterministic and can be characterized by its mean and its variance. In case of a multi-objective optimization, one can either combine all objective functions into one single objective function by a weighted sum, or, alternatively, keep the different objectives which usually leads to a so-called Pareto front. These are different designs, where, if one compares design x on the front with design y on the front, x is not better than y in all objectives.
The polynomial chaos method (PCM) based uncertainty quantification (UQ) approach offers a large potential for non-deterministic simulations. In PCM stochastic variables with different distributions can be handled both with intrusive and non-intrusive approaches. The properties of the input random variables and the output stochastic solution can be described in terms of the mean, variance, higher order moments and probability density functions. The polynomial chaos methodology was originally formulated by Wiener [1] in an intrusive framework and was further expanded by Xiu and Karniadakis [2]. Several non-intrusive polynomial chaos methods (NIPCM) were also developed during recent years [3, 4, 5, 6]. NIPCM are sampling based methods and can be implemented easily into any in-house and commercial CFD code. For these reasons, in this work, quadrature based NIPCM is used.

For optimization sampling based random search algorithms (gradient free, such as Genetic algorithm) [7, 8] and gradient based methods [9, 10] are common methodologies. In this work, a gradient based approach is combined with the non-intrusive polynomial chaos for robust optimization in CFD applications. Gradient based techniques are nowadays used widely for optimization with a large number of design variables. The adjoint system of equations is similar to the governing flow equations so the numerical schemes for solving the flow equations can be used for the adjoint solution. The adjoint methods are computationally very efficient as the computational cost to compute the gradient is roughly same as the cost of a CFD solution and is independent on the number of shape parameters.

In the present work, a new framework for robust optimization is presented. The main objective is to combine the non-intrusive polynomial chaos uncertainty quantification method with adjoint methods for robust optimization and apply it to the optimal shape design of the RAE2822 airfoil in the transonic flow under operational uncertainties. The Mach number and angle of attack are considered as uniformly distributed random variables in the design process.

2 Adjoint based robust optimization

Adjoint methods are nowadays used to optimize a wide range of problems with large number of design variables efficiently. Many optimization methods such as Newton’s method, Steepest descent, Gauss-Newton, Nonlinear least squares, rely on the gradients of the objective function. The gradient based method is surely not computationally inexpensive. It allows fast convergence to the optimum once close to its basin of attraction. On the other hand, compared to evolutionary algorithms, it can more easily get stuck in a local optimum. It also allows to explore only a part of the design space, in the neighborhood of the starting design point.

2.1 The Adjoint method

In aeronautical and CFD applications, the adjoint methods have long been considered as a preferable choice for gradient based optimization. In a design optimization problem, an objective function \( J(U, \alpha) \) is a function of the state variables \( U \) and the design variables \( \alpha \). The adjoint method aims at computing the gradient with respect to the design variables \( \alpha \). The total derivatives \( \frac{dJ}{d\alpha} \) can be written as:

\[
G = \frac{dJ}{d\alpha} = \frac{\partial J}{\partial \alpha} + \frac{\partial J}{\partial U} \frac{dU}{d\alpha}
\]  

(1)

The partial derivatives in the above equation can be evaluated directly by varying the design variables and reevaluating the objective function in the numerator. However, evaluation of term \( \frac{dU}{d\alpha} \) requires the solution of the governing equations. If \( R(U, \alpha) = 0 \) represents the flow equations of an aerodynamic problem, the total derivative of the flow equations with respect to the design variables \( \alpha \) can be expressed as:

\[
\frac{dR}{d\alpha} = \frac{\partial R}{\partial \alpha} + \frac{\partial R}{\partial U} \frac{dU}{d\alpha} = 0
\]  

(2)
or

\[ \frac{dU}{d\alpha} = - \left( \frac{\partial R}{\partial U} \right)^{-1} \frac{\partial R}{\partial \alpha} \]  

(3)

By combining equation (1) and (3), we obtain the following expression for the total gradient:

\[ G = \frac{dJ}{d\alpha} = \frac{\partial J}{\partial \alpha} - \left( \frac{\partial R}{\partial U} \right)^{-1} \frac{\partial R}{\partial \alpha} \]  

(4)

Now let assume that the Lagrangian multiplier \( \lambda \) satisfies the following linear equation:

\[ \left( \frac{\partial R}{\partial U} \right)^T \lambda = \left( \frac{\partial J}{\partial U} \right)^T \]  

(5)

which is also known as the adjoint equation. Combining equations (4) and (5) one finally obtains the following expression for the gradient \( G \):

\[ G = \frac{dJ}{d\alpha} = \frac{\partial J}{\partial \alpha} - \lambda^T \frac{\partial R}{\partial \alpha} \]  

(6)

2.2 Non-intrusive polynomial chaos method

In polynomial chaos methods a complete set of polynomials is used up to a given order of PC approximation. The stochastic solution \( u(x; \xi) \) in terms of PC expression is written as:

\[ u(x; \xi) = \sum_{i=0}^{P} u_i(x) \psi_i(\xi) \]  

(7)

where \( u_i \)'s are deterministic coefficients and \( \psi_i \) are orthogonal polynomials. The total number of terms \( P + 1 \) depends on the highest order of polynomial \( p \) and on the number of random dimensions \( n_s \) as: \( P + 1 = \frac{(n_s+p)!}{n_s!p!} \).

From the deterministic PC coefficients, \( u_o(x) \) is the mean and the variance of the solution can be calculated as:

\[ \sigma^2 = \sum_{i=1}^{P} u_i^2(x) < \psi_i^2(\xi) > \]  

(8)

To compute the polynomial coefficients \( u_i \) from the solution samples \( u^k \), the quadrature method is used. For a second order PC approximation and two random variables \( \xi = \{ \xi_1, \xi_2 \} \), the multidimensional Legendre polynomials are:

\[ \psi_0(\xi) = 1 \]
\[ \psi_1(\xi) = \xi_1 \]
\[ \psi_2(\xi) = \xi_2 \]
\[ \psi_3(\xi) = \frac{1}{2}(3\xi_1^2 - 1) \]
\[ \psi_4(\xi) = \xi_1\xi_2 \]
\[ \psi_5(\xi) = \frac{1}{2}(3\xi_2^2 - 1) \]  

(9)
2.3 Adjoint method for stochastic applications

In stochastic applications, the objective function \( J \) will also be stochastic and can be written in terms of polynomial chaos expansion as:

\[
J = \sum_{i=0}^{P} J_i \psi_i
\]  

(10)

where the mean value of \( J \) is \( \bar{J} = J_0 \) and the variance of \( J \) is \( \sigma_J^2 = \sum_{i=1}^{P} J_i^2 <\psi_i^2> \).

The gradient of the objective function \( J \) with respect to the design variable can be expressed as:

\[
G = \nabla J = \sum_{i=0}^{P} \nabla J_i \psi_i = \sum_{i=0}^{P} \frac{dJ_i}{d\alpha} \psi_i = \sum_{i=0}^{P} G_i \psi_i
\]  

(11)

In stochastic applications, the stochastic objective function is usually written as the weighted sum of its statistical moments. Therefore, a single objective deterministic optimization becomes a multi-objective optimization in the stochastic case. Considering a multi-objective optimization, a new objective function can be defined as a linear combination of the mean and the variance of original objective function as:

\[
I = k_1 \bar{J} + k_2 \sigma_J
\]  

(12)

where \( k_1 + k_2 = 1 \). In order to optimize (or minimize) the objective function \( I \) using gradient based methods, one needs to compute the gradient of the objective function \( I \) with respect to the design variable as:

\[
\nabla I = k_1 \nabla \bar{J} + k_2 \nabla \sigma_J
\]  

(13)

From equation (11):

\[
\nabla \bar{J} = \nabla J_0 = G_0
\]  

(14)

where \( \nabla \bar{J} \) is the gradient of the mean and \( G_0 \) is the mean of the objective gradient. The gradient of the variance can be expressed as:

\[
\nabla \sigma_J^2 = \sum_{i=1}^{P} \nabla J_i^2 <\psi_i^2>
\]  

(15)

or

\[
2\sigma_J \nabla \sigma_J = 2 \sum_{i=1}^{P} J_i G_i <\psi_i^2>
\]  

(16)

The above equation can further simplified as:

\[
\nabla \sigma_J = \frac{1}{\sigma_J} \sum_{i=1}^{P} J_i G_i <\psi_i^2>
\]  

(17)
3 Application to the RAE2822 airfoil

3.1 Test case description

Here, the robust design methodology is applied to one of the basic test case of the UMRIDA project, the RAE2822 airfoil. The airfoil geometry is described in Cook et al [11]. The nominal flow conditions correspond to Mach number $M_{\infty} = 0.729$, angle of attack $\alpha = 2.31^\circ$ and Reynolds number $Re_{\infty} = 6.5 \times 10^6$. The Mach number and the angle of attack are considered as uncertain parameters with standard deviations of the Mach number and angle of attack, $\sigma_M = 0.01$ and $\sigma_\alpha = 0.4^\circ$ respectively. Both uncertain inputs are considered as uniformly distributed parameters. For uncertainty quantification, the quadrature based second order polynomial method is used in each iteration of the optimization process. Here the adjoint solver and the flow solver of SU2 (an open source CFD solver) are coupled with the polynomial chaos methods for the optimal shape design of the airfoil. The objectives considered are the mean drag coefficient and its variance. For robust optimization, the shape of the airfoil is parameterized and modified using a set of Hicks-Henne bump functions. Hicks-Henne bump functions recently received popularity in modeling small perturbations in many shape optimization problems. The bump function is applied to the baseline airfoil to modify the shape of the airfoil. A total of 38 Hicks-Henne bump functions are applied to the upper and the lower surface of the airfoil at 5%, 10%..., 95% of the chord length. In this way, 38 coefficients of the bump functions are considered as the design variables. The optimization procedure is performed using the sequential least square programming (SLSQP) algorithm [12].

3.2 Deterministic solution

For the RAE2822 airfoil, the unstructured mesh with 13,937 cells, used for the CFD simulation is shown in Figure 1. A verified and validated solver SU2 is used for deterministic CFD solutions. The Spalart-Allmaras (SA) model is used for modeling of turbulence. In addition to solving the RANS equations, the adjoint equations can be also solved in SU2 for sensitivity analysis and optimization purposes. In Figure 2, the pressure coefficient for a test case (free-stream conditions : $M_{\infty}=0.729$, $\alpha = 2.31^\circ$, $Re_{\infty} = 6.5 \times 10^6$, Pressure = 101325 pa, Temperature=288.15 K) is depicted. In Figure 2, the pressure coefficient distribution obtained with the SU2 solver is also compared with experimental results. It can be observed that the numerical predictions of the $C_p$ using SA turbulence modeling are in close agreement with the measurements.

Figure 1: RAE2822: Computational domain (left) and zoomed mesh near to airfoil (right)
In a shape optimization problem, we try to optimize the chosen objective function with the changes in shape. Using the adjoint approach, the gradient of the objective function with respect to the deformation in the geometry can be computed. The adjoint solver of SU2 is used to compute the surface sensitivity. In Figure 3, the surface sensitivity of the objective function (Drag) is shown due to infinitesimal deformation on the RAE2822 airfoil surface in the local normal direction. It is seen that the surface sensitivity is large near the leading and the trailing edges of the airfoil. The adjoint solver of SU2 computes these values at each node of the airfoil surface.

3.3 Shape parameterization

The Hicks-Henne bump functions are applied to the baseline airfoil to modify the shape of the airfoil. The Hicks-Henne functions are defined as:

\[ f(x) = \left[ \sin(\pi x \log_{0.5} t) \right]^{12}; 0 \leq x \leq 1 \]

where, \( t_1 \) is the x location of the maximum and \( t_2 \) is the width of the bump. In Figure 4, the Hicks-Henne bump functions for \( t_2 = 10 \) and \( t_1 = 0.05, 0.10, 0.15, ... 0.95 \) are shown. The bump functions reach maximum at the given values of \( t_1 = 0.05, 0.10, 0.15, ... 0.95 \).

The modified airfoil shape can be expressed as a weighted sum of sin functions (Hickes-Hinne):
A set of weights \((a_1, a_2...a_N)\) is employed to control the magnitude of the shape functions, where \(a_i\) are design variables and \(N\) is the total number of design variables.

### 3.4 SQLSP for optimization

Sequential least square programming (SQLSP) or sequential quadrature programming method is considered as one of the most powerful gradient based algorithms. SQLSP method finds the minimum of a constrained objective function of multiple variables.

Matlab subroutine "fmincon" of python script "fmin_sqlqp" can be used to use SQLSP algorithm for optimization. SQLSP solves problems of the form:

\[
\begin{align*}
\min \ f(x) \\
\text{subjected to :} \\
\text{inequalities: } g_i(x) \leq 0; \ i = 1,...,m \\
\text{equalities: } h_j(x) = 0; \ j = 1,...,n \\
\text{bounds: } l_b \leq x \leq u_b
\end{align*}
\]

The SQLQP algorithm optimizes the objective function for a given accuracy or a fixed number of iteration. The SQLQP algorithm implements the method of Han and Powell [13] with BFGS update. The basic idea of SQLQP is to create and solve a subproblem with quadratic objective function to the Lagrangian in each iterations.

### 3.5 Robust optimization

In this section, the initial geometry of the RAE2822 is taken for the shape optimization in order to minimize the drag (objective function). An in-built gradient based optimizer SQLQP in SU2 framework combines the flow solver, the adjoint solver and the mesh deformation tools to minimize the objective function. The surface sensitivities (change in the objective function due to an infinitesimal deformation
Figure 5: Optimization history for the mean and the standard deviation for the 2 cases

Figure 6: Airfoil shape: RAE baseline and Optimized

on the airfoil surface) are used to compute the gradient of the objective function with respect to the design variables. By using a simple chain rule, the surface sensitivities (computed with adjoint solver) are projected into the design space. In the robust optimization process, the objective function (a weighted sum of the mean and its standard deviation) and the gradient of the objective function (as equation 13) are provided to the optimizer. By changing the weights, the results for two different test cases are obtained. The first case is where a higher weight is given to the mean (i.e. $K_1 = 0.9$ and $K_2 = 0.1$) and for the second test case, more weight is given to the standard deviation (i.e. $K_1 = 0.1$ and $K_2 = 0.9$). In Figure 5, the optimization history of the mean and the standard deviation for the two test cases is
shown. From the optimization history, one can see that for both test cases, the mean and the variances of the drag coefficients are optimized after a few iterations only. In Table 1, the mean and the standard deviation of the drag coefficient are shown for the baseline RAE2822, deterministic optimized airfoil and for the robust design of the two test cases. From Table 1, it can be seen that for the case 1 (where more weight is given to the mean) the optimized mean drag is lower than all other designs. Similarly for case 2 (where more weight is given to the standard deviation), the standard deviation is lower than all other designs. When the uncertainties are not considered in the design process, the mean drag coefficient is reduced by approximately 15% and the coefficients of variance by approximately 25% (see Table 1). When the uncertainties are included in the design process (case 1 and case 2), the coefficients of variance are reduced by approximately 50%, making the final designs robust with respect to uncertainties. In Figure 6, the airfoil designs for all cases, i.e. the baseline RAE2822, optimal shape (without uncertainties) and robust shape for both the test cases (under uncertainties) are shown. It can be seen that during the optimization process, mostly the area near to shock region is deformed. Two different airfoil designs (for the two cases under uncertainties) are obtained. The shape of these airfoils are completely different from those without uncertainties.

<table>
<thead>
<tr>
<th></th>
<th>RAE2822</th>
<th>Optimized</th>
<th>Robust design, K=(0.9,0.1)</th>
<th>Robust design , K=(0.1,0.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.45729679e-02</td>
<td>1.24683603e-02</td>
<td>1.20657193e-02</td>
<td>1.27927945e-02</td>
</tr>
<tr>
<td>Std</td>
<td>3.88569535e-03</td>
<td>2.51665720e-03</td>
<td>1.47263413e-03</td>
<td>1.33563551e-03</td>
</tr>
<tr>
<td>CoV</td>
<td>26.66% mean</td>
<td>20.18% mean</td>
<td>12.20% mean</td>
<td>10.44% of mean</td>
</tr>
</tbody>
</table>

Table 1: Mean and std of the drag coefficient for baseline RAE, optimized and robust airfoils

4 Conclusion

In this work, a new robust optimization method is introduced and applied to the optimal shape design of the RAE2822 airfoil. A non-intrusive polynomial chaos based uncertainty quantification method is combined with the flow solver and the adjoint solver of SU2. Two optimal designs are obtained by considering two different points on the Pareto front (by changing the weights of the mean and the variance). It is shown that the case where a higher weight is given to the mean, the mean value of the drag is lower than all other designs. Similarly when a higher weight is given to the variance, the variance of the drag was found lower than all other designs.

5 Acknowledgements

This work was partly done within the UMRIDA project which is supported by the European Unions Seventh Framework Programme for research, technological development and demonstration under grant agreement no. ACP3-GA-2013-605036. The authors gratefully acknowledge this EU funding.

6 References


