Response Variability of a Tuned Liquid Column Damper Applied to a Wind Turbine

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Abstract

Passive energy dissipation systems encompass a range of materials and devices for enhancing damping. They can be used both for natural hazard mitigation and for rehabilitation of aging or deficient structures. Among the current passive energy dissipation systems, tuned liquid column damper (TLCD), a class of passive control that utilizes liquid in a “U” shape reservoir to control structural vibration of the primary system, has been widely researched in a variety of applications. Due to the variability in temperature and other parameters the absorber properties can change and interfere in the optimized operation of the damper. The aim is to simulate the response of the system to a variability in the damper parameter considering uncertainties associated with the TLCD damping to analyze the absorber behavior. The method consist of insert uncertainties in the damper element by constructing a probabilistic model from the Maximum Entropy principle. Then, a Monte Carlo simulation is made to describe the implications of this variability in the system response. The uncertainty analysis in important to describe the impact in the damper performance and help the design of the optimum damper considering this variability. Finally, a numerical example considering a wind turbine tower as a primary system is made to analyze its response through a wind spectrum.

Keywords: TLCD, Vibration Absorber, Damper optimization, Random Uncertainty, Stochastic Analysis, Monte Carlo Simulation

1 Introduction

Remarkable progress in the technology used in wind turbines has been made over the past years; advances in the field of structural and dynamic analysis allow the creation of larger and more efficient wind turbines. However, higher and slender structures poses challenges concerning its integrity in relation to the dynamic loads from wind, ocean waves or earthquakes. Serious efforts have been undertaken to develop the concept of vibration control of wind turbines.

Vibration control has raised various technical studies in recent years [1–4]. Additionally, different types of devices have encounter vibration control applications in wind turbines [4–6]. In particular, among the energy dissipation systems, Tuned Liquid Column Dampers (TLCDs) are emerging in several specialized publications and has become a good option due to its relatively low cost and good efficiency.

The main goal is reduce vibration in wind turbines using a tuned liquid column damper. As shown in Figure 1, the TLCD operates based on the movement of the liquid column. The column may have different shapes, particularly in this paper, the TLCD has a “U” shape. The TLCD requires no extra mechanism such as springs or joints, besides that, its geometry may vary according to design needs, making them very versatile devices. While the apparent simplicity of the system, the damping is dependent on the amplitude of the liquid, and therefore the dynamics of TLCD is nonlinear which brings some mathematical complications to the model.

In this paper, the aim is to simulate the response of the system to a variability in the damper parameter. The variability in temperature and others parameters can cause changes in the damper properties and interfere in the optimized operation of the damper, hence, it is important to study this variability in the vibration response of the model. The method consist of insert uncertainties in the damper element and constructing a probabilistic model from the Maximum Entropy principle and finally a Monte Carlo simulation is made to describe the implications of this variability in the system. The uncertainty analysis in important to describe the impact in the damper performance and help the design of the optimum damper considering this variability.
An important element of any experiment is the ability to quantify uncertainty in the results that can be attributed to random variables and in the description of dynamic structures.

Random vibration corresponds to oscillation of the mechanical system subject to random variation in the input either temporal or spatial. Their study is particularly important because all real physical systems are subject to some random dynamic effect during its service life and many of them will fail due to the exposure effects. Studies about random vibration are in search of explanations for such phenomena aiming to predict the system response characteristics as a way to aid in mechanical design.

To increase the credibility of the model, these uncertainties need to be modeled using the theories of probability and statistics. The study of randomness associated with mechanical systems was introduced in the early 20th century, however, only the external loading was considered random leaving uncertainties related to the model unconsidered [7].

Random vibrations have been observed for centuries in the effects of the winds, earthquakes, waves and other natural conditions although their study at the light of mathematics is quite recent. Einstein realized what can be considered the first analysis of random vibration when he considered the Brownian motion of particles suspended in a liquid in a study published in 1905. Many other studies whose results would be used to explain the mechanical systems under random vibration were performed in the decades following. In 1930, Norber Weiner formally defined the spectral density of a stationary random process that even today has a central role in the quantitative description of stationary random processes. It was only in 1950 that the random vibration theory would be addressed directly because of the need to create a theory that foresaw precisely structural response to the noise of jet engines and environments induced by the missile launch [8].

To quantify uncertainties in the model of dynamic structures, random variables need to be associated with the system along with their probability density function. Building the probability density function that best represents the physical problem is not trivial and requires experimental data to assist in its construction. One way around this problem is to associate the random variable a Gaussian probability density function. However this procedure is not always advisable as it may lead to results physically impossible.

The Maximum Entropy Principle can be used to construct the probability density function of the random variable uncertainties of the model. The principle consist in using only the information available to build possible probability density functions and then search for the function with maximum entropy (or uncertainty). This method avoids using misinformation in the construction of model ranging from the physics of the problem.

There are two paths that can be followed in the model uncertainties. random scalar variable, when scalar parameters are modeled with uncertainty and random matrix variable when model uncertainty is included by random matrices. The first case is indicated for uncertain data model while the latter is suitable for uncertainties in the model also known as nonparametric approach [7]. The scope of this paper will be limited to the random scalar variable method.

This paper is organized in 4 sections, section 2 introduces both deterministic and stochastic model of the TLCD. The Monte Carlo approach is describe in section 3. Finally, section 4 presents an example of a numerical response.

2 Stochastic Dynamical System

2.1 Deterministic model

Consider the TLCD model mounted structure as sketched in Fig. 2 and 1. The idealization for the structure is acceptable because the support has negligible mass, thus, it is possible to approach the shear-frame system as a one degree of freedom model with stiffness and equivalent damping.

The equation describing the motion of the fluid in the TLCD is given by

$$\rho Al\ddot{u}(t) + \frac{1}{2} \rho A \xi |\dot{u}(t)| \dot{u}(t) + 2 \rho Agu(t) = -\rho Ab\ddot{x}(t), \quad (1)$$

where $u(t)$ is the displacement of fluid function, $x(t)$ is the displacement of the primary system function, $\rho$ is the fluid density, $\xi$ is the head loss coefficient, $A$ is the cross section area of the column, $b$ and $l$
are the horizontal and total length of the column respectively and $g$ is the gravity constant. It can be observed that the TLCD mass is given by $m_a = \rho A l$, the TLCD damping is $c_a = \frac{1}{2} \rho A \xi |\dot{u}(t)|$ and the TLCD stiffness is given by $k_a = \rho A g$. The natural frequency of oscillation in the column can be obtained by $\omega_a = \sqrt{2g/l}$.

The equation of motion of the primary structure is given by

\[(m_e + m_a)\ddot{x}(t) + \rho Ab\ddot{u}(t) + c_e \dot{x}(t) + k_e x(t) = F(t),\]  

(2)

where the parameter $m_e$ is the structure mass, $k_e$ the structure stiffness, $c_e$ the structure damping and $F(t)$ the external force. Thus, combining Eq.(1) and Eq.(2). The equation of motion in matrix form can be written as

\[
\begin{bmatrix}
    m_e + m_a & \alpha m_a \\
    \alpha m_a & m_a \\
\end{bmatrix}
\begin{bmatrix}
    \ddot{x} \\
    \ddot{u} \\
\end{bmatrix}
+
\begin{bmatrix}
    c_e & 0 \\
    0 & c_a \\
\end{bmatrix}
\begin{bmatrix}
    \dot{x} \\
    \dot{u} \\
\end{bmatrix}
+
\begin{bmatrix}
    k_e & 0 \\
    0 & k_a \\
\end{bmatrix}
\begin{bmatrix}
    x \\
    u \\
\end{bmatrix}
=
\begin{bmatrix}
    F(t) \\
    0 \\
\end{bmatrix},
\]

(3)

where $\alpha = b/l$ is the dimensionless length ratio. The condition presented by Eq.(3) is needed to ensure that the liquid in the column do not spill water and consequently change its damping characteristic.

Equation (3) can also be written with the mass matrix in its dimensionless form, given by

\[
\begin{bmatrix}
    1 + \mu & \alpha \mu \\
    \alpha \mu & 1 \\
\end{bmatrix}
\begin{bmatrix}
    \ddot{x} \\
    \ddot{u} \\
\end{bmatrix}
+
\begin{bmatrix}
    2\omega_e \xi \omega_e \\
    \xi |\dot{u}(t)| \omega_e \\
\end{bmatrix}
\begin{bmatrix}
    \dot{x} \\
    \dot{u} \\
\end{bmatrix}
+
\begin{bmatrix}
    \omega_e^2 & 0 \\
    0 & \omega_a^2 \\
\end{bmatrix}
\begin{bmatrix}
    x \\
    u \\
\end{bmatrix}
=
\begin{bmatrix}
    \frac{F(t)}{m_e} \\
    0 \\
\end{bmatrix},
\]

(4)
where $\zeta_c$ and $\omega_c$ are the damping ratio and natural frequency of the structure, respectively. The dimensionless parameters mass ratio $\mu$ and tuning ratio $\gamma$ are defined as

$$\mu = \frac{m_a}{m_e}, \quad \gamma = \frac{\omega_a}{\omega_e}. \quad (5)$$

The nonlinear nature of the damping requires the determination of an equivalent value to the damping coefficient. Roberts and Spanos [9] proposed a procedure to estimate the optimum value of the damping coefficient utilizing the statistical linearization method. It is possible to express the error between the nonlinear system with the equivalent linear system as $\epsilon = (1/2)\rho A\xi |\dot{u}| - c_{eq} \dot{u}$, where the value of the equivalent damping $c_{eq}$ can be obtained by minimizing the standard deviation of the error value, namely $E\{\epsilon^2\}$. Assuming that the liquid velocity has Gaussian form, the expression for the equivalent damping is given as [9]

$$c_{eq} = \sqrt{\frac{2}{\pi}} \rho A \xi \sigma_u = 2 \omega_a \zeta_a, \quad (6)$$

where $\sigma_u$ is the standard deviation of the fluid velocity. Therefore, the equivalent damping approached by statistical linearization $c_{eq}$ can replace the nonlinear value $c_a$ in Eq. (3). The equivalent damping can also be expressed as a function of the absorber damping ratio $\zeta_a$ in order to make the optimization calculation more convenient.

The equation of motion can be written in the Fourier domain and one can obtain the the frequency response function (FRF) for the two degree of freedom as follows,

$$\hat{H}(i\omega) = \frac{\dot{x}}{\dot{F}}, \quad \hat{G}(i\omega) = \frac{\ddot{u}}{\dot{F}}, \quad (7)$$

where $\omega$ stands for the driving frequency, $\dot{x}$ denotes the Fourier transform of $x$, $\ddot{u}$ denotes the Fourier transform of $u$ and $\dot{F}$ denotes the Fourier transform of $F$, then it follows

$$\hat{H}(i\omega) = -\Delta \mu \alpha (i\omega)^2 + (i\omega)^2 + \zeta_c \omega_a (i\omega) + \omega_a^2 \left( (i\omega)^2 + 2 \zeta_c \omega_a (i\omega) + \omega_a^2 \right) - (i\omega)^2 \alpha^2 \mu, \quad (8)$$

$$\hat{G}(i\omega) = -\alpha (i\omega)^2 + \Delta \left( (i\omega)^2 (1 + \mu) + 2 \zeta_c \omega_a (i\omega) + \omega_a^2 \right) \left( (i\omega)^2 + 2 \zeta_c \omega_a (i\omega) + \omega_a^2 \right) - (i\omega)^4 \alpha^2 \mu, \quad (9)$$

where $\Delta$ indicates the reference in the analysis of the system, when $\Delta = 1$, one has base excitation and $x$ is a relative displacement. When $\Delta = 0$ one has excitation in the primary system and $x$ is the absolute displacement.

### 2.2 Uncertainty Model

The uncertainty analysis is important in order to be able to describe how the temperature and other parameters may impact the damper performance and improve the design reliability considering the optimum damping and its variability. The probabilistic parameter is assumed to be the viscous damping coefficient $\zeta_a$. It can be noted that the mass and stiffness are assumed to be deterministic. The transfer function $\hat{H}$ is nonlinear with respect to the probabilistic parameters [10].

First, the probability distribution function will be constructed using the Maximum Entropy principle [11]. By relying only to the information available, it is possible to obtain the optimum probabilistic model using the one with maximum entropy (uncertainty).

The parameter considered as uncertain is the TLCD damping ratio $\zeta_a$ and considering the random variable $C$ associated to this parameter. The basic available information are the mean reduced model, the positive-definiteness of the random variable and the existence of second-order moments, in other words: (1) the support of the probability density function is $[0, +\infty]$, (2) the mean value is assumed to be known, $E\{C\} = \bar{C}$ and (3) the condition $E\{\ln(C)\} < +\infty$, which implies that zero is a repulsive
value [12]. The probability density function $p_C$ has to verify the following constraint equations [13]

$$\int_{-\infty}^{+\infty} p_C(c) dc = 1,$$

$$\int_{-\infty}^{+\infty} cp_C(c) dc = \mathbb{C},$$

$$\int_{-\infty}^{+\infty} \ln(C) p_C(c) dc < +\infty,$$

applying the Maximum Entropy Principle yields the following probability density function [12]

$$p_C(c) = 1_{[0, +\infty]}(c) \frac{1}{\mathbb{C}} \left( \frac{1}{\delta_C^2} \right) \frac{1}{\Gamma(1/\delta_C^2)} \left( \frac{1}{\mathbb{C}} \right)^{\frac{1}{\delta_C^2} - 1} \frac{-c}{e^{\frac{1}{\mathbb{C}}}} e^{\frac{c}{\mathbb{C}}},$$

where $\delta_C = \sigma_C/\mathbb{C}$ is the coefficient of dispersion of the random variable $C$ and $\sigma_C$ is the standard deviation of $C$. It can be verified that $C$ is a second-order random variable and that $E\{1/C^2\} < +\infty$. The Gamma function is defined as

$$\Gamma(1/\delta_C^2) = \int_{0}^{+\infty} t^{1/\delta_C^2 - 1} e^{-t} dt, \quad 1/\delta_C^2 > 0.$$ 

3 Monte Carlo Sampling

The Monte Carlo method is a class of computational techniques based on synthetic generation of random variables in order to deduce the implications for the probability distribution. In probabilistic simulations, we must ensure that the probability density function of the random variable has significant physical meaning reason why we use the Maximum Entropy principle.

Simulation convergence criterion is given by [7]

$$\text{conv}(n_s) = \frac{1}{n_s} \sum_{j=1}^{n_s} \int_B \left\| H_j(\theta, \omega) - \hat{H}(\omega) \right\|^2 d\omega,$$

where $\|H_j(\theta, \omega)\|$ is the stochastic system response calculated for the $\theta$ realization, $\|\hat{H}(\omega)\|$ is the mean stochastic system response.

The deterministic model is obtained by using the $\mathbb{C}$ itself. The value of $\mathbb{C}$ is determined by an optimization method developed by Kareem and Yalla [14] which for a white-noise excitation and considering undamped primary system, it can be expressed as

$$\mathbb{C} = \alpha \sqrt{\frac{2\mu \left( \alpha^2 \mu^2 - \mu - 1 \right)}{2 \left( \alpha^2 \mu^2 - 4\mu - 2\mu^2 - 2 \right)},}$$

where $\alpha = 0.9$ and $\mu = 0.05$, obtaining $\mathbb{C} = 0.0965$.

4 Results from Numerical Example

Murtagh et al. [15, 16] present a simplified model of wind turbine tower as a cantilever beam that can be reduced to a one degree of freedom (1DoF) model with equivalent mass and stiffness parameters.

The wind turbine has a steel tower with 60 m hub height, 3 m width and 0.015 m thickness. The modulus of elasticity is $E = 2.1 \times 10^{11} \text{ N/m}^2$ and density of the steel is $\rho = 7,850 \text{ kg/m}^3$. The rotor mass is $M = 19,876 \text{ kg}$. Using the dimensionless parameter length ratio $\alpha = 0.9$ and $\nu = 0.1$, one can obtain $\omega_c = 3.6450$ and $\zeta_c = 0.0018$. 

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Figure 3: All realizations of FRF, the Mean value, deterministic and 5% and 95% percentile for $\delta_C = 5.5733$.

Figure 4: Mean square convergence for $\delta_C = 3.9409$. 
Figure 5: Mean value, deterministic value and 5% and 95% percentile of FRF for different $\delta_C$ (a) 3.9409 (b) 4.5506 (c) 5.0877 (d) 5.5733 (e) 6.0199 (f) 6.4355.
Figure 3 shows all realizations of a Monte Carlo simulation considering 3000 realizations and for $\delta_C = 5.5733$, the mean value and the deterministic value. The deterministic value was calculated using the mean damping ratio which was obtained from a statistical linearization considering white noise excitation and Gaussian behavior from Eq.16. It can be noticed that the mean value of all realizations does not coincide with the deterministic value. Besides that, one can see that for some realizations, the system behaves as a one degree of freedom as the damping value from the absorber becomes negligible. However, these does not occur as often since the mean value is far from these realizations.

The typical convergence rate is shown in Figure 4, occurring for above 1,000 MC samples.

Since the value of the standard deviation is not known, different values of $\delta_C$ are used to analyze the behavior of the FRF. Figure 5 shows the FRF and it respective confidence regions for different values of the coefficients of dispersion $\delta_C$. The confidence intervals take into consideration the 95% more frequently results and the 5% less frequently result according to the probability density function. As the coefficient of dispersion increases the limits of the confidence intervals get thinner. From the amplitude between the confidence intervals, we can notice a amplitude range from around 10 m. This range seems pretty substantial even if one consider the size of a wind turbine.

Small values of $\delta_C$ means that the random variable associated to the damping ratio has a small standard deviation since the mean value is fixed. It is worth notice that the value of the coefficients of dispersion must not be chosen arbitrarily, in fact, its value has to be within a certain boundary that satisfy the condition $E(C^2) < +\infty$. When the value of the coefficients of dispersion is big the response limits are to wide to make any satisfactory result [17].

5 Concluding Remarks

In this paper, a stochastic analysis in the damping parameter was proposed. The variability in temperature and other parameter may cause changes in the damper properties which can interfere in the optimized operation of the damper, hence, it is important to study this variability in the vibration response of the model. The method consisted of insert uncertainties in the damper element, construct the probabilistic model from the Maximum Entropy principle and finally perform a Monte Carlo simulation to describe the implications of this variability in the system. From the Frequency Response Function the amplitude range of the wind turbine tower vary around 10 m which seems pretty substantial.

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